BEEs01/ESGI123
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Collapse of paperboard packages

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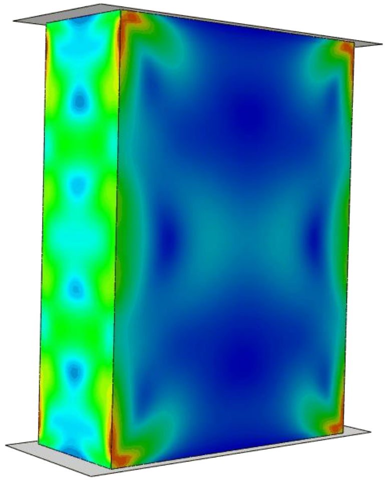
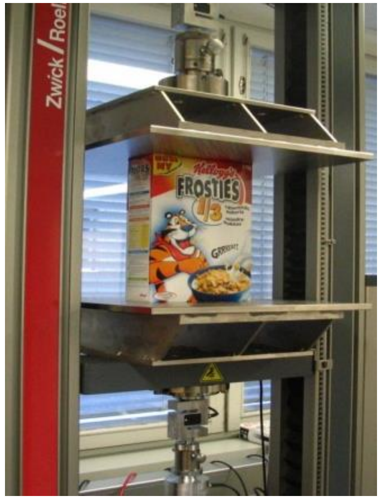
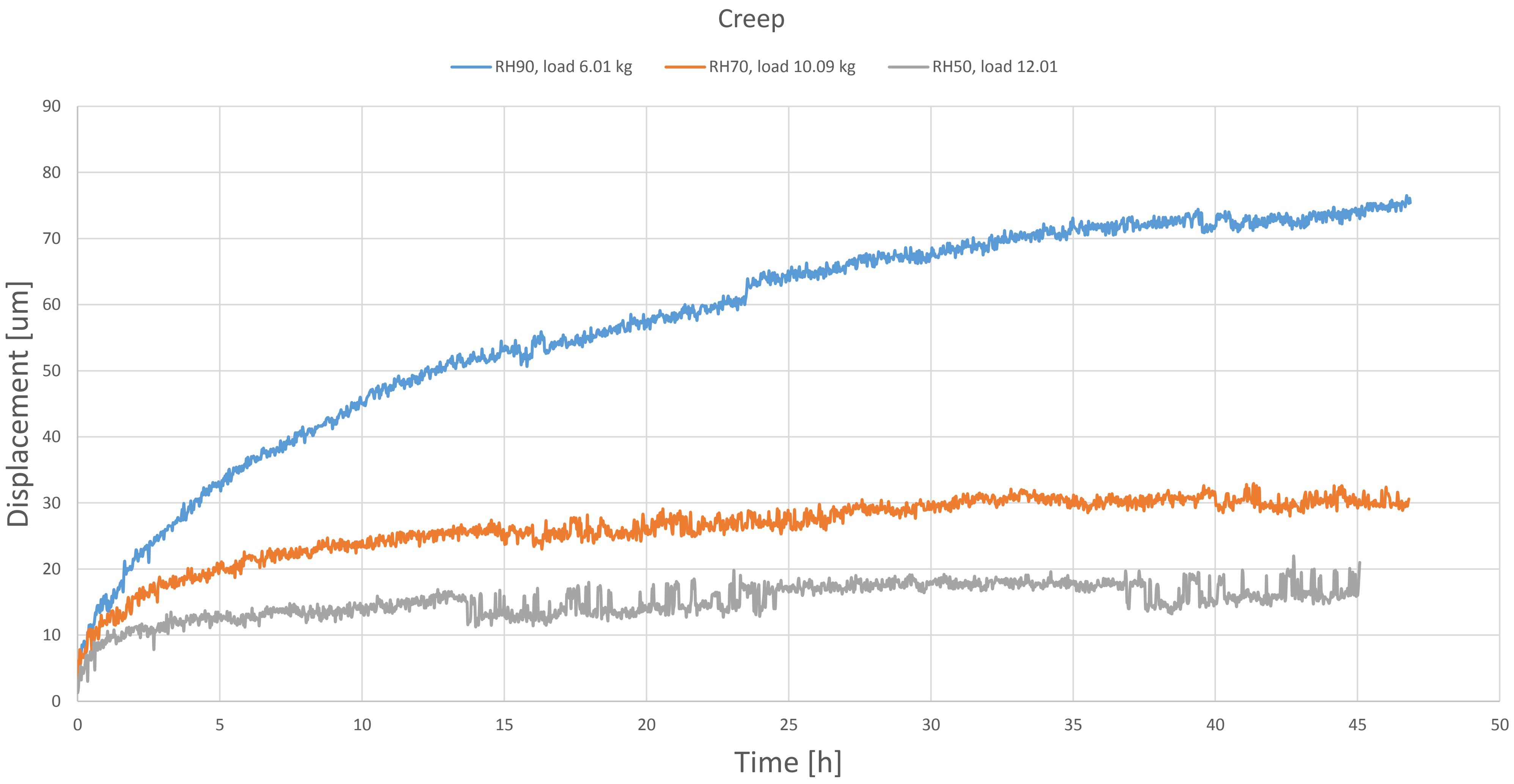
Tasks

1. Determine a time- and moisture dependent material model for paperboard
2. Implement the material model to finite element software
3. Use finite element simulations to investigate creep behavior of paperboard packages under cyclic humidity conditions






Creep
RH90, load 6.01 kg
RH70, load 10.09 kg
RH50, load 12.01
Displacement [um]
0
10
20
30
40
50
60
70
80
90
Time [h]
0
5
10
15
20
25
30
35
40
45
50

1D-model without moisture

- Strain-stress relation

$$\varepsilon(t) = \int_0^t D(t-s)\sigma'(s)ds;$$

- Idea:

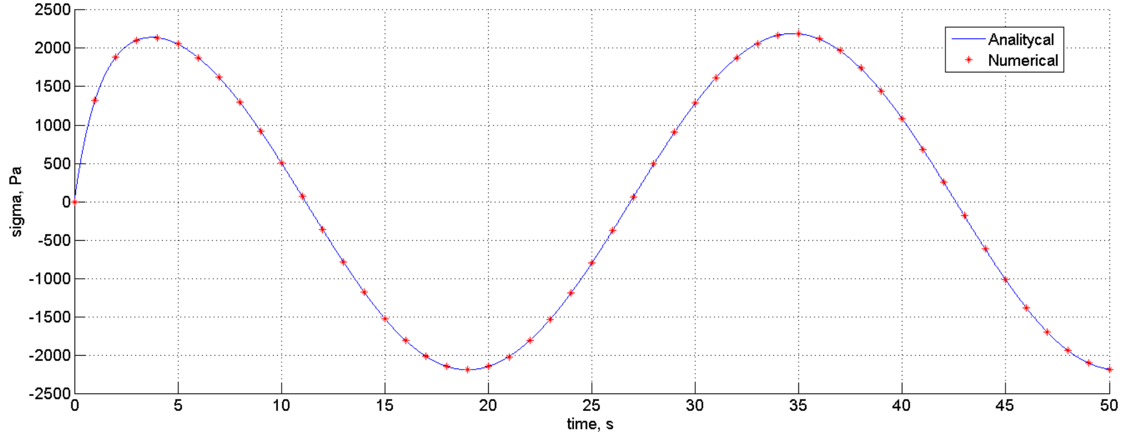
$$D(t) = D_0 + \sum_{i=1}^{n} D_i(1 - e^{-t/\tau_i});$$

$$q_i(t) = \int_0^t e^{-\frac{t-s}{\tau_i}} \sigma'(s)ds;$$

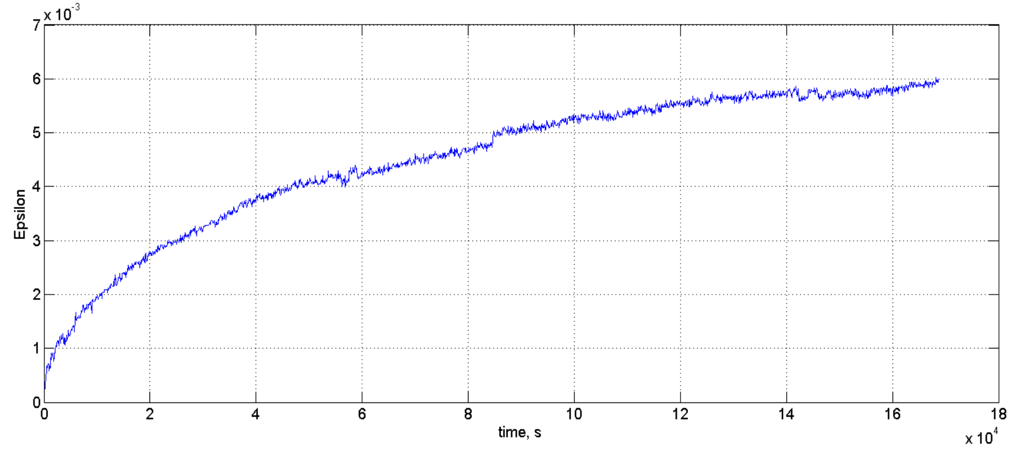
$$\Delta\sigma_{n+1} = \frac{\Delta\varepsilon_{n+1} + \sum_{i=1}^{n} D_i(\frac{\tau_i}{\tau_i + \Delta t} - 1)q_{n,i}}{D_0 + \sum_{i=1}^{n} D_i - \sum_{i=1}^{n} D_i \frac{\tau_i}{\tau_i + \Delta t}}$$

$$\Delta q_{n+1,i} = (\frac{\tau_i}{\tau_i + \Delta t} - 1)q_{n,i} + \frac{\tau_i}{\tau_i + \Delta t}\Delta\sigma_{n+1};$$

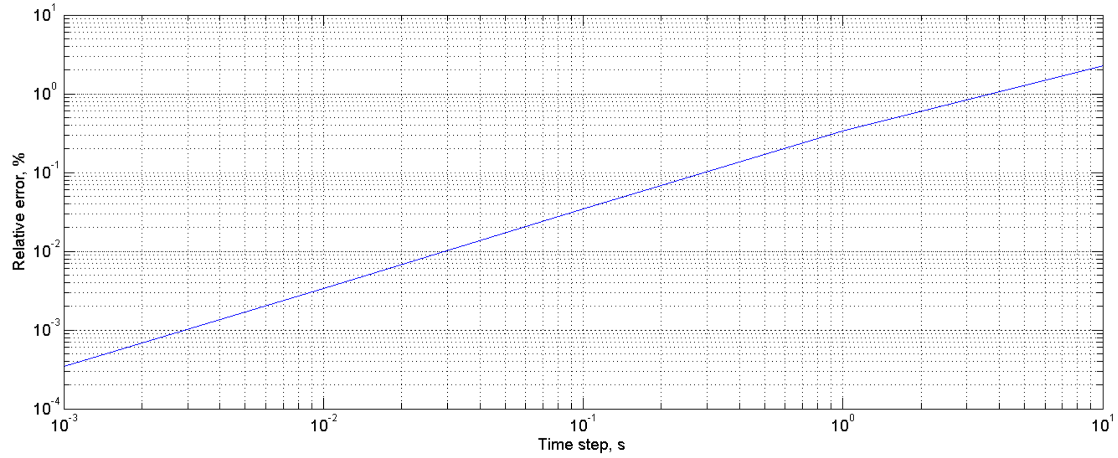
ε=A*sin(Bt)

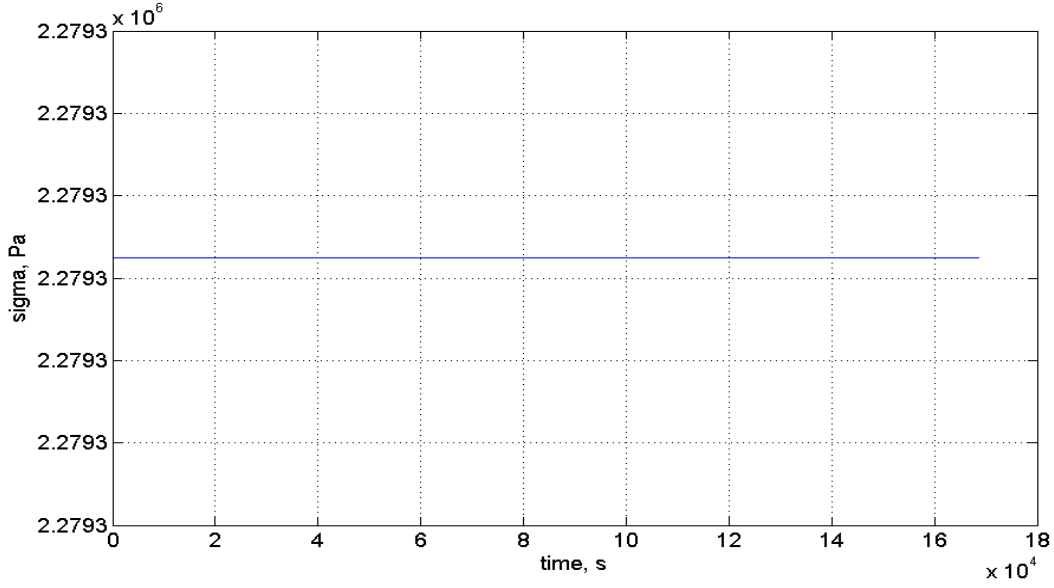


ε from experimental data



Convergence





1D-model with moisture

Strain-stress relation
$$\varepsilon(t) = \int_0^t D(\psi(t) - \psi(s))\sigma'(s)ds;$$
$$\psi(t) = \int_0^t \frac{d\eta}{a(c(\eta))};$$

First step
$$\sigma(t) = \sigma_0 H(t) \Longrightarrow \sigma'(t) = \sigma_0 \delta(t)$$

Strain
$$\varepsilon(t) = \sigma_0 D(t) \rightsquigarrow \text{DATA}$$

Model
$$D(a,t) = D_0 + \sum_{i=1}^{n} D_i \left(1 - e^{-t/a\tau_i}\right)$$

1D-model with moisture

Strain-stress relation

$$\varepsilon(t)=\int_0^t D(\psi(t)-\psi(s))\sigma'(s)ds;$$

$$\psi(t)=\int_0^t \frac{d\eta}{a(c(\eta))};$$

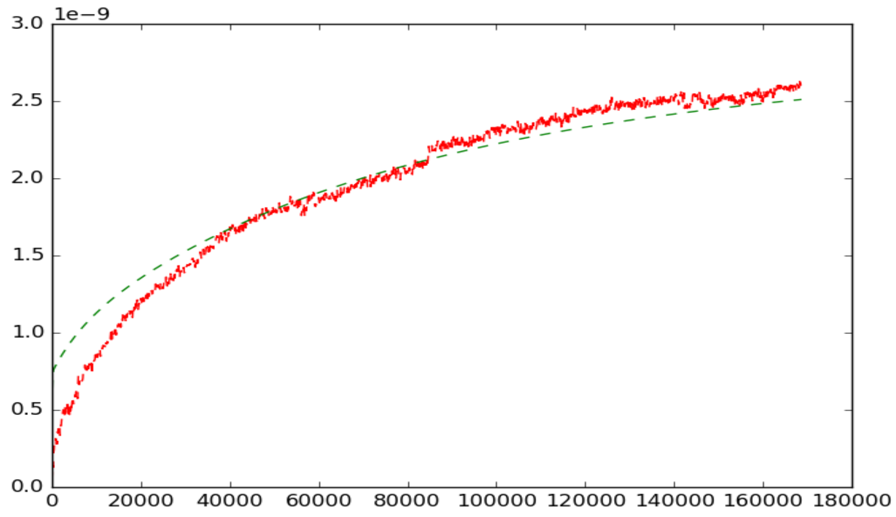
Known: D, τ, ε, init. cond.

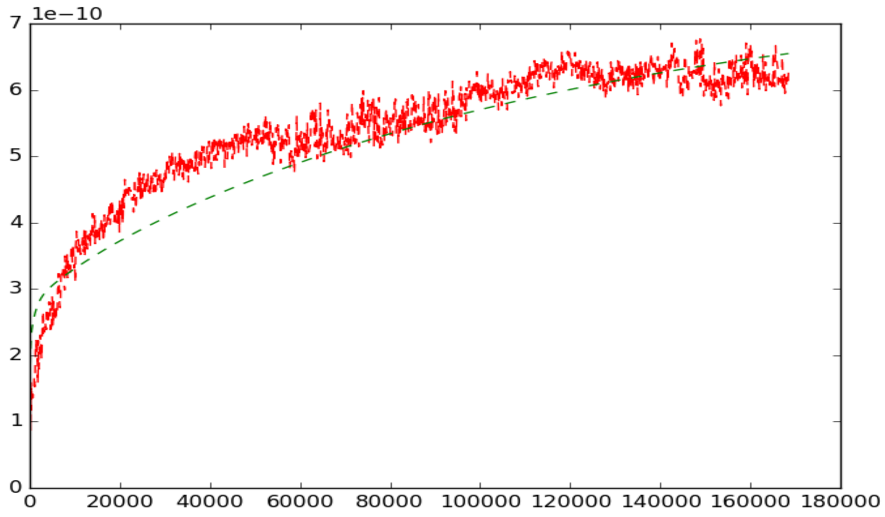
Unknown σ, ψ

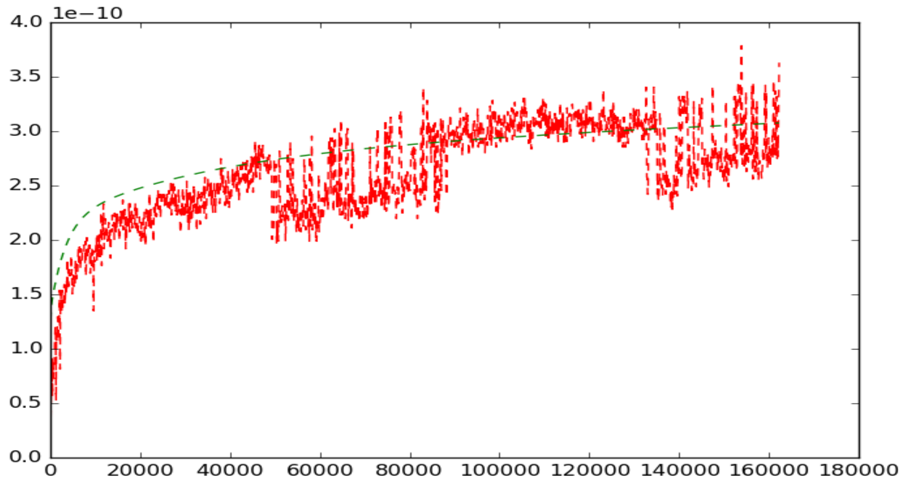
First step $\sigma(t)=\sigma_0 H(t)\Longrightarrow \sigma'(t)=\sigma_0\delta(t)$

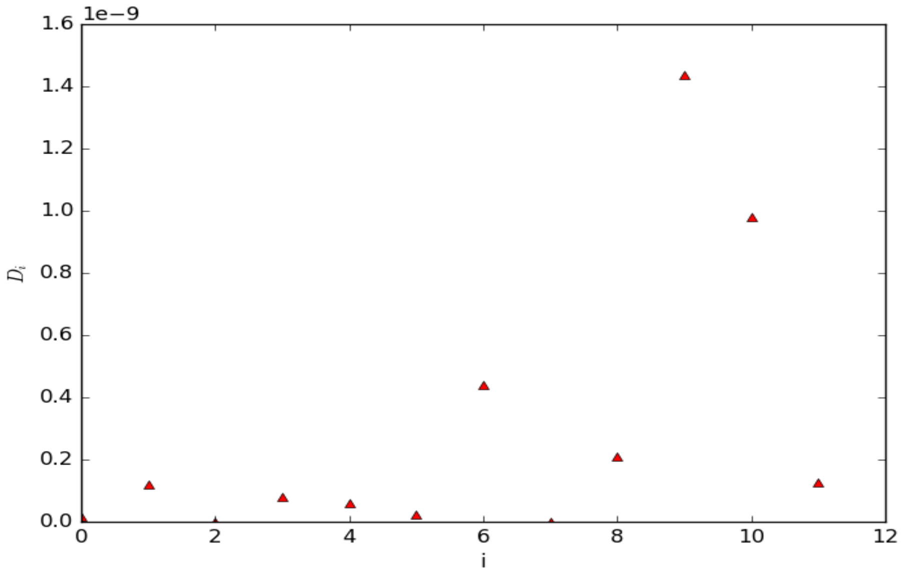
Strain $\varepsilon(t)=\sigma_0 D(t)$ ⇝ DATA

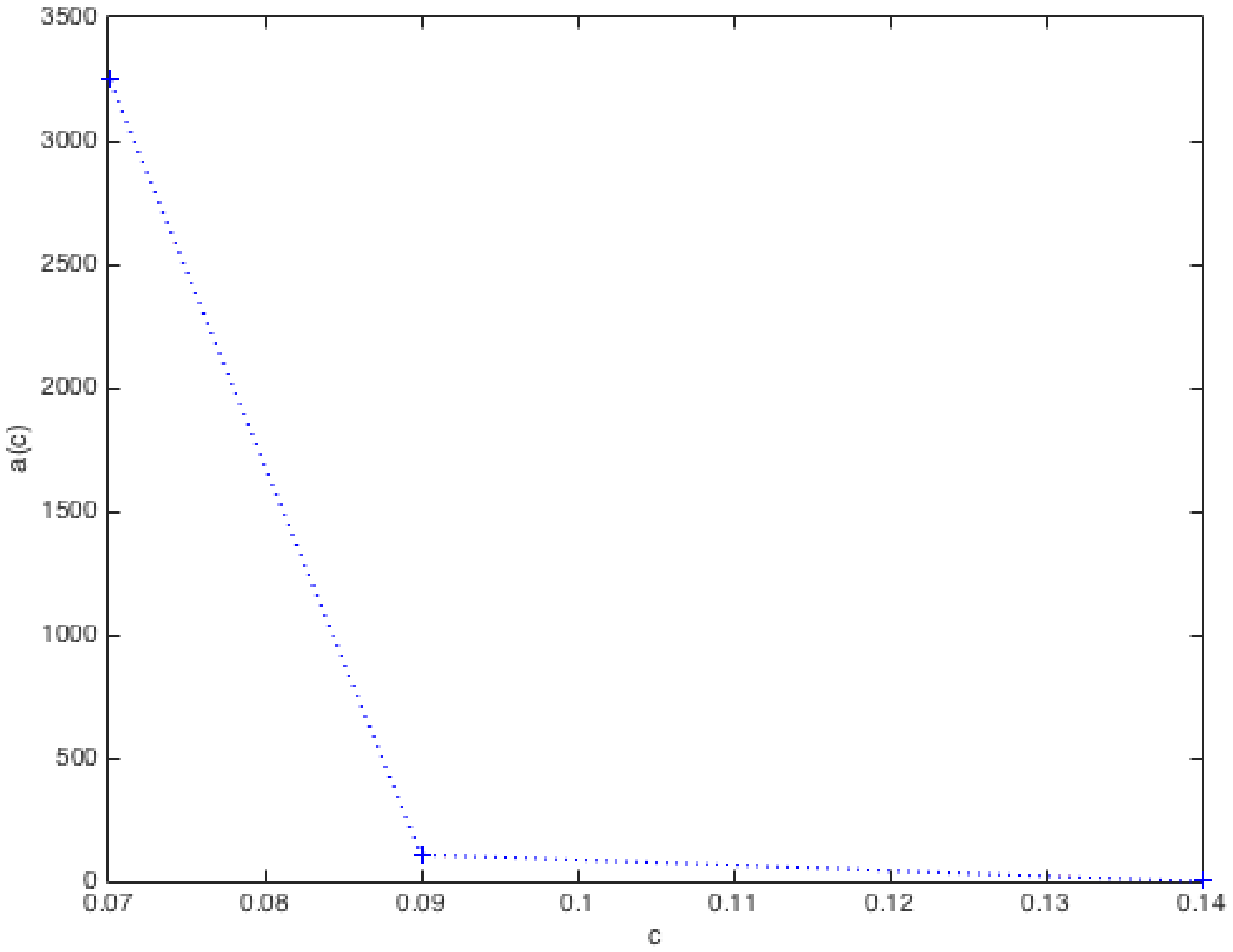
Model $D_j(a_j,t)=D_0+\sum_{i=1}^{n} D_i\left(1-e^{-t/a_j\tau_i}\right)$


1e−9
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0 20000 40000 60000 80000 100000 120000 140000 160000 180000


1e−10
7
6
5
4
3
2
1
0
0 20000 40000 60000 80000 100000 120000 140000 160000 180000


1e−10
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0 20000 40000 60000 80000 100000 120000 140000 160000 180000


1e−9
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
D_i
0 2 4 6 8 10 12
i



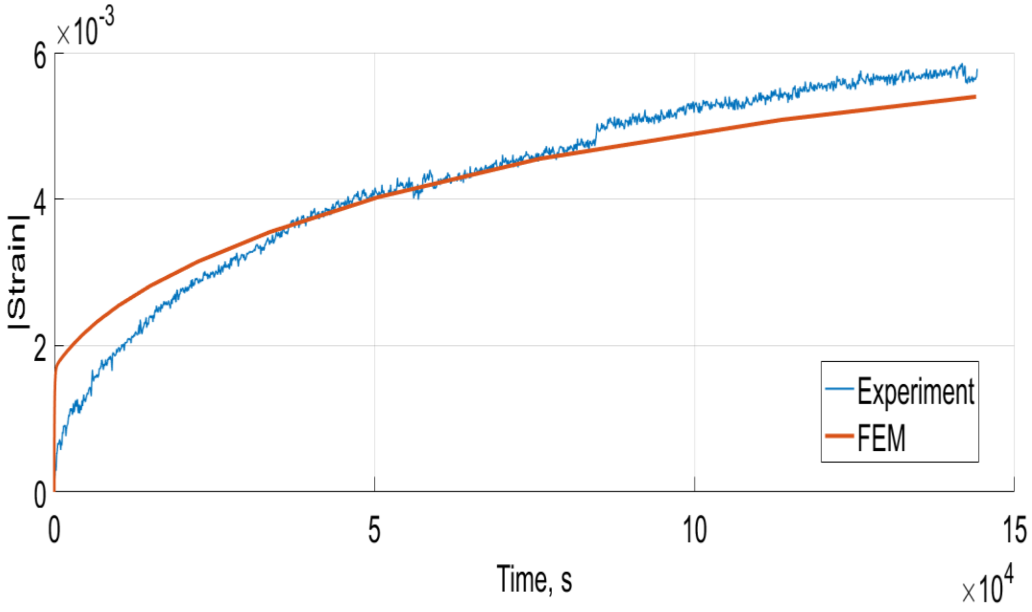
If c < 0.09

a = $f_1(c) = k_1 * c + b_1$

Else a = $f_2(c) = k_2 * c + b_2$

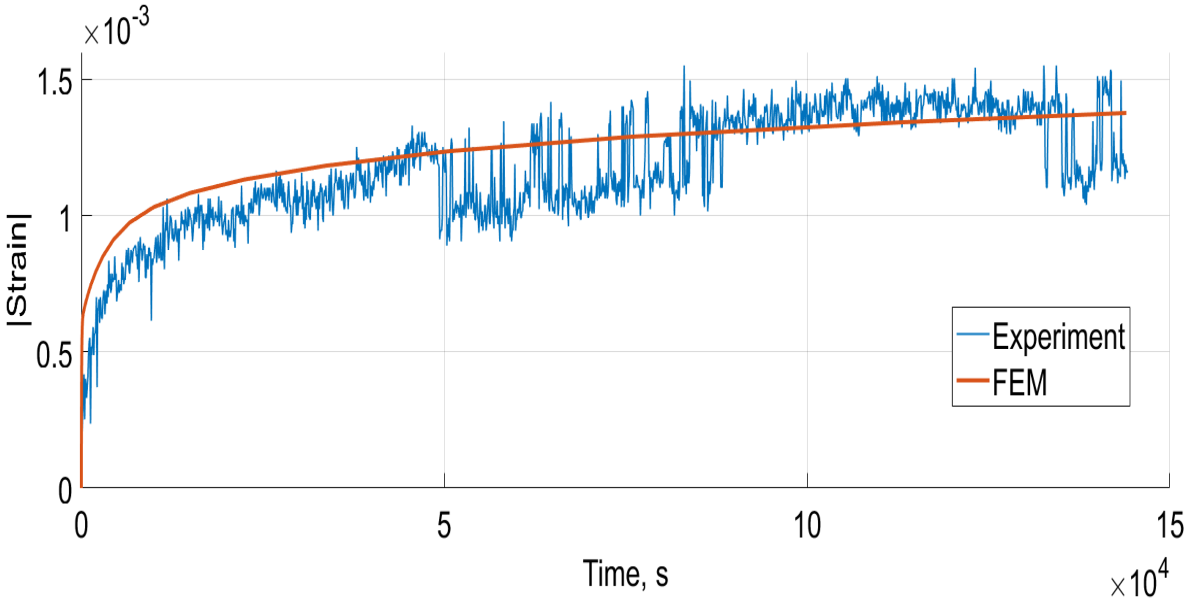
1D model

Rh – relative humidity

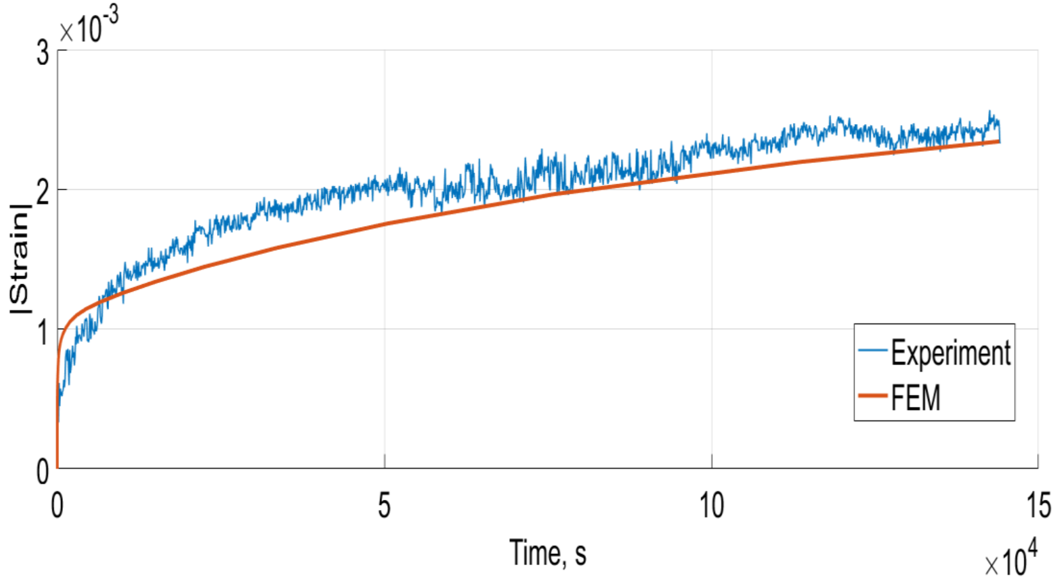


Rh=90%

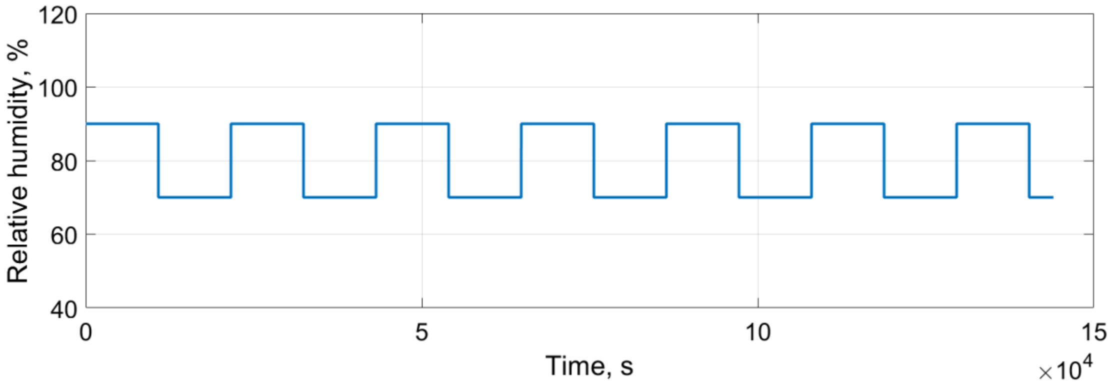
Rh=70%



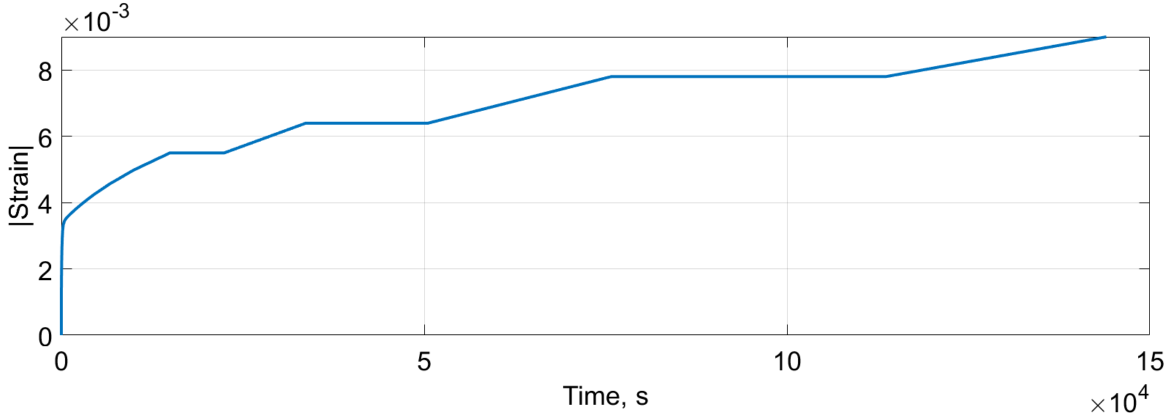
Rh=50%



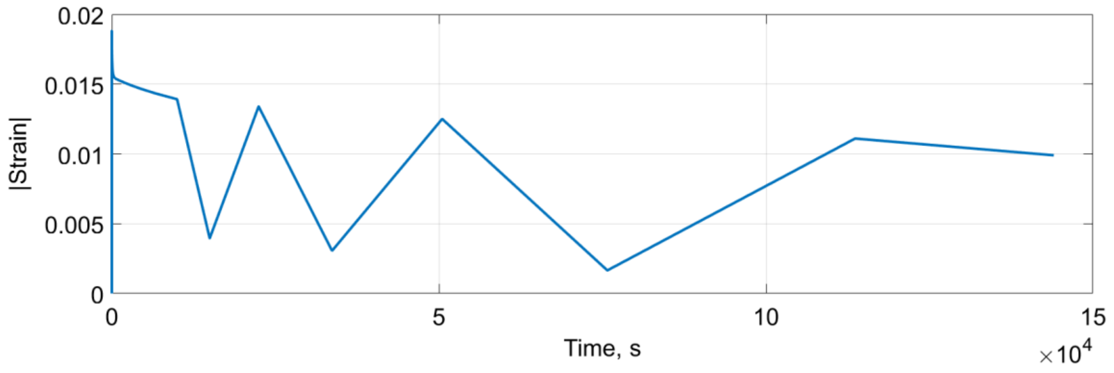
1D model



Rh=70-90%



Rh=70-90%
(with expansion)



2D-model

<u>*Known:*</u> D, ψ, τ, ε, a, init. conds.

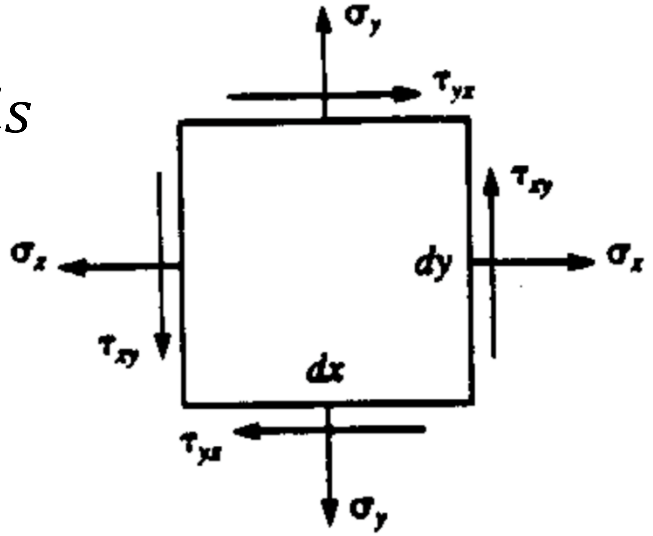
<u>*Unknown:*</u> σ

- Classical Planar Stress Model

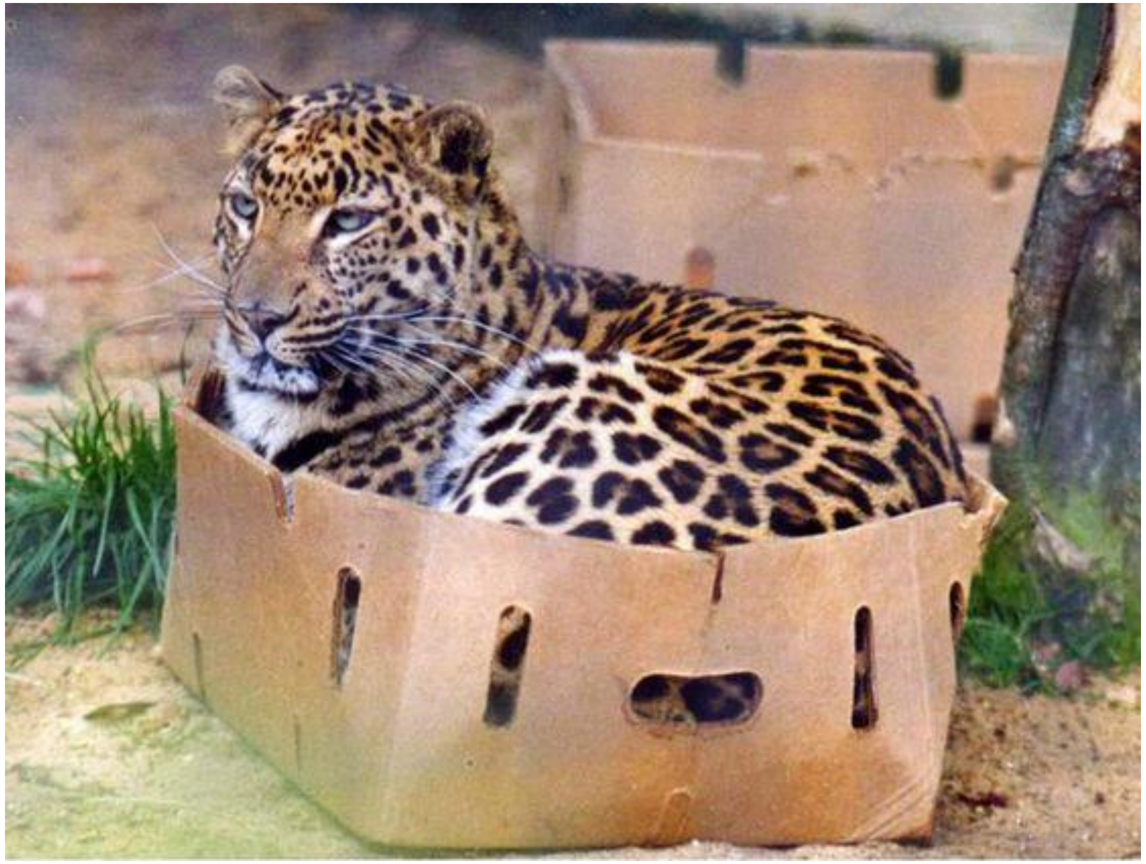
$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{pmatrix} = D \begin{pmatrix} 1 & -\upsilon & 0 \\ -\upsilon & 1 & 0 \\ 0 & 0 & (1+\upsilon) \end{pmatrix} \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{pmatrix}$$

- Time-dependent model with moisture

$$\begin{pmatrix} \varepsilon_x(t) \\ \varepsilon_y(t) \\ \varepsilon_{xy}(t) \end{pmatrix} = \int_0^t D(\psi(t) - \psi(s)) \begin{pmatrix} 1 & -\upsilon & 0 \\ -\upsilon & 1 & 0 \\ 0 & 0 & (1+\upsilon) \end{pmatrix} \begin{pmatrix} \sigma_x(s) \\ \sigma_y(s) \\ \sigma_{xy}(s) \end{pmatrix} ds$$



Thank you for your attention



Appendix 1

Without moisture

$$\varepsilon(t) = \int_0^t D(t-\tau)\sigma'(\tau)d\tau;$$

$$D(t) = D_0 + \sum_{i=1}^{n} D_i(1 - e^{-t/\tau_i});$$

$$q_i(t) = \int_0^t e^{-\frac{t-\tau}{\tau_i}} \sigma'(\tau)d\tau;$$

$$\Delta\sigma_{n+1} = \frac{\Delta\varepsilon_{n+1} + \sum_{i=1}^{n} D_i(\frac{\tau_i}{\tau_i + \Delta t} - 1)q_{n,i}}{D_0 + \sum_{i=1}^{n} D_i - \sum_{i=1}^{n} D_i \frac{\tau_i}{\tau_i + \Delta t}}$$

$$\Delta q_{n+1,i} = (\frac{\tau_i}{\tau_i + \Delta t} - 1)q_{n,i} + \frac{\tau_i}{\tau_i + \Delta t}\Delta\sigma_{n+1};$$

With moisture

$$\varepsilon(t) = \int_0^t D(\psi(t) - \psi(\tau))\sigma'(\tau)d\tau;$$

$$\psi(t) = \int_0^t \frac{d\eta}{a(c(\eta))};$$

$$\Delta\sigma_{n+1} = \frac{\Delta\varepsilon_{n+1} + \sum_{i=1}^{n} D_i(\frac{\tau_i a_{n+1}}{\tau_i a_{n+1} + \Delta t} - 1)q_{n,i}}{D_0 + \sum_{i=1}^{n} D_i - \sum_{i=1}^{n} D_i \frac{\tau_i a_{n+1}}{\tau_i a_{n+1} + \Delta t}}$$

$$\Delta q_{n+1,i} = (\frac{\tau_i a_{n+1}}{\tau_i a_{n+1} + \Delta t} - 1)q_{n,i} + \frac{\tau_i a_{n+1}}{\tau_i a_{n+1} + \Delta t}\Delta\sigma_{n+1};$$

Appendix 2

Optimization with The Nelder–Mead method (or downhill simplex method or amoeba method) is used for approximation of curves

$$Err_j = \min_{D_0, D_i, a_k} \left(\frac{D_{app}}{D_{data}} - 1\right), \qquad i = \overline{1,n};\ j = \overline{1,3};\ k = \overline{1,3}$$

$$Err_{main} = \frac{\sum_{j=1}^{3} b_j Err_j}{3}, b_j \ - weight\ coefficients$$